COLUMBIA | ENGINEERING
The Fu Foundation School of Engineering and Applied Science

ELEN E3106/4106 Lecture 5

Carriers: Temperature Dependence, Drift, Mobility, and Resistance

Outline

- Temperature effects
- Compensation
- Space charge neutrality
- Mobility & scattering
- Drift velocity & current
- Resistivity

Assignments:
Reading: Streetman and Banerjee §3.3.3-3.3.4, 3.4
Homework 2 due Friday Sept 19th by 5pm

Recap of carrier concentrations

- Recall from Lecture 4, we learned how to get e⁻ and h⁺ concentrations at:
  - Any ___________________
  - Any ___________________
  - Any ___________________
- We saw that in thermal equilibrium, $np \;= $ ___
- And, and $n_i^2 = \; n_0 p_0 \; = N_c N_v e^{-\frac{E_g}{kT}}$, even when $n_0 \; \neq p_0$
- Given the density of states,

$$N_c = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{\frac{3}{2}} \qquad N_v = 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{\frac{3}{2}}$$

- And we can conveniently find the carrier concentrations,

$$n_0 = n_i e^{(E_F - E_i)/kT} \qquad p_0 = n_i e^{-(E_i - E_F)/kT}$$

Temperature Dependency of *Intrinsic* Carrier Concentrations

- At any given temperature, *T*:

$$n_i = \sqrt{N_c N_v} e^{-\frac{E_g}{2kT}}$$

- But it can also be written,

$$n_i(T) = 2\left(\frac{2\pi kT}{h^2}\right)^{3/2} (m_n^* m_p^*)^{3/4} e^{-E_g/2kT}$$

- What does this tell us?

- n_i is very temperature-sensitive! In silicon,
  - While T = 300 → 330 K (10% increase)
  - n_i = ~10^{10} → ~10^{11} cm^{-3} (____ increase)

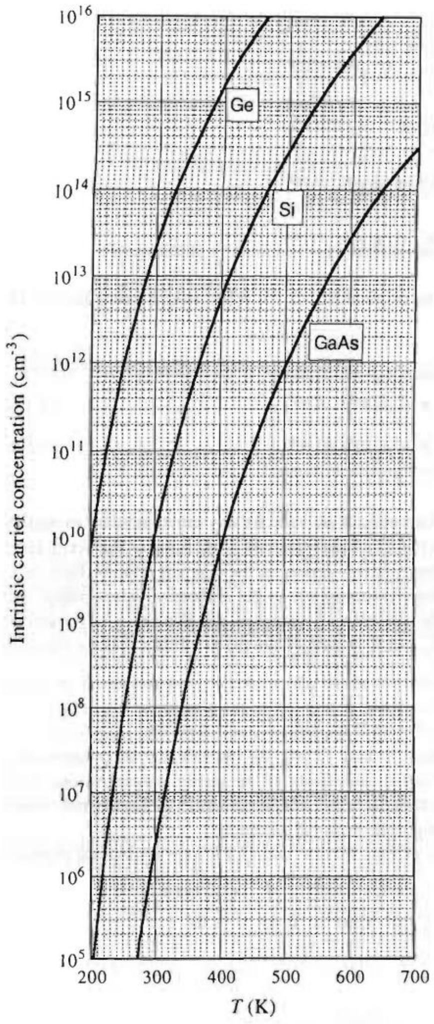
Sources: E. Pop ECE 340 Slides, Textbook

Visualizing Intrinsic Carrier Concentrations

- Plot $\log_{10}$ of n_i vs. T

- What do we expect?

- What is this plot neglecting?

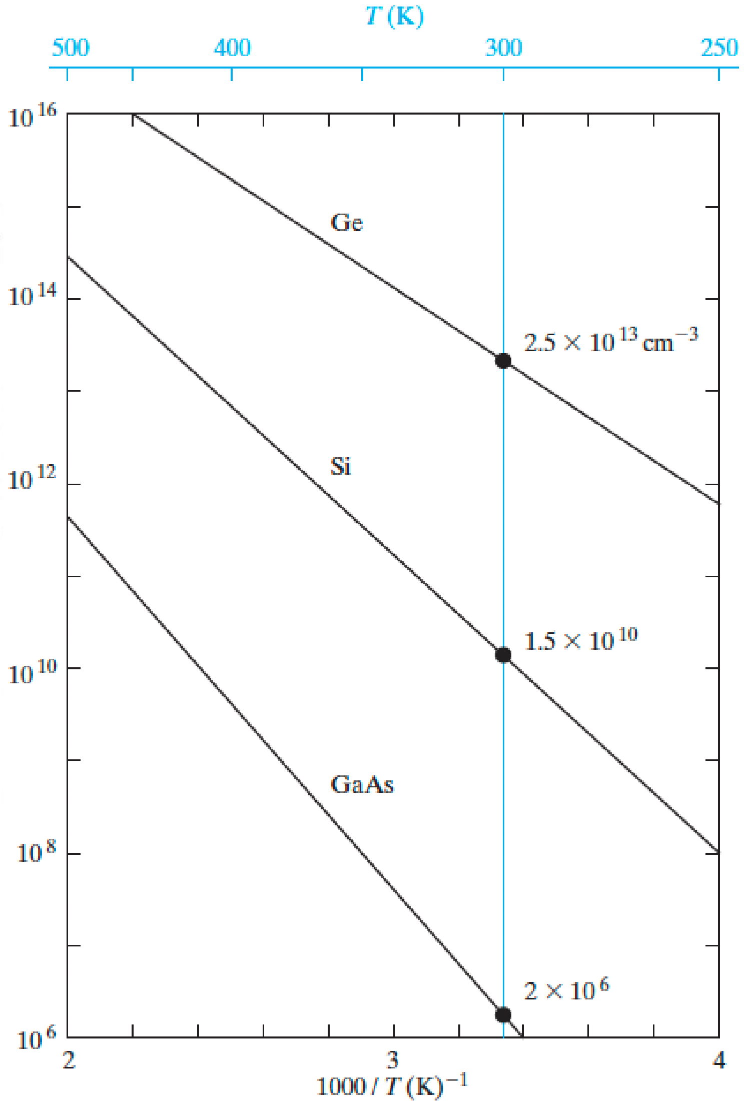


Si

T(°C)	n_i(cm^{-3})
0	8.86×10^8
5	1.44×10^9
10	2.30×10^9
15	3.62×10^9
20	5.62×10^9
25	8.60×10^9
30	1.30×10^{10}
35	1.93×10^{10}
40	2.85×10^{10}
45	4.15×10^{10}
50	5.97×10^{10}
300 K	1.00×10^{10}

GaAs

T(°C)	n_i(cm^{-3})
0	1.02×10^5
5	1.89×10^5
10	3.45×10^5
15	6.15×10^5
20	1.08×10^6
25	1.85×10^6
30	3.13×10^6
35	5.20×10^6
40	8.51×10^6
45	1.37×10^7
50	2.18×10^7
300 K	2.25×10^6



Sources: E. Pop ECE 340 Slides, Textbook

Problem: Calculating Intrinsic Carrier Concentration

- If we know n_i and *T*, we have two unknowns: __________ and the carrier concentration. If we know one of the two, we can solve for the other:

$$n_0 = n_i e^{(E_F - E_i)/kT}$$
$$p_0 = n_i e^{(E_i - E_F)/kT}$$

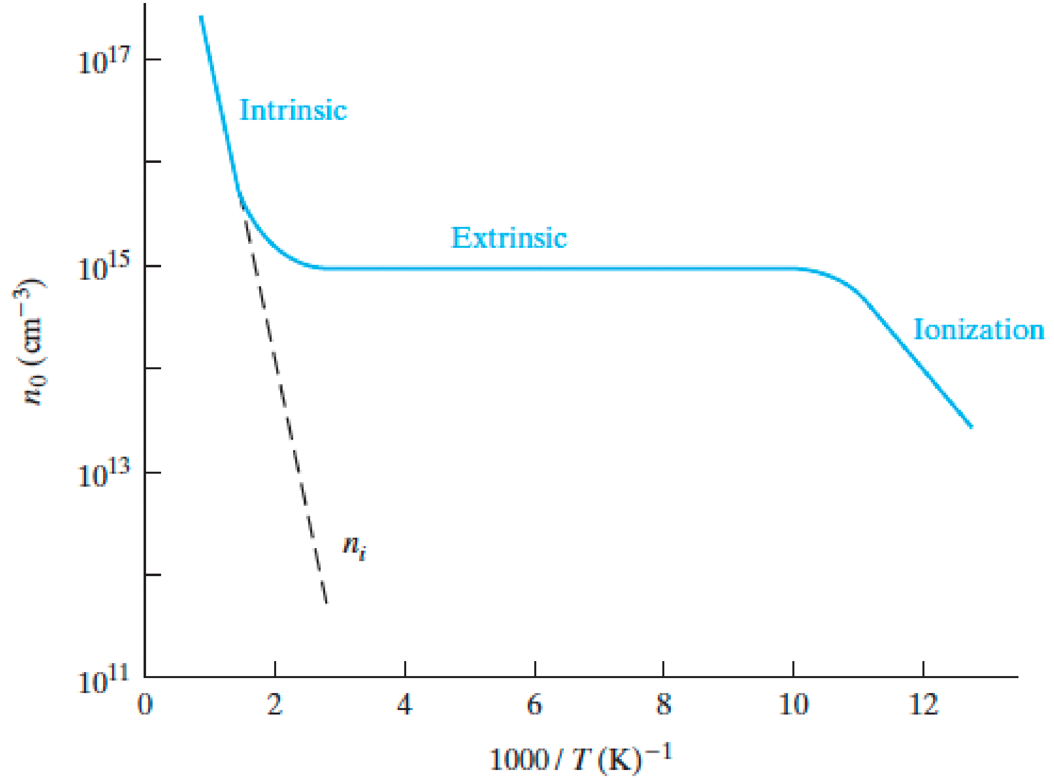
- Calculate and show position of the Fermi level in doped Ge (10^{16} cm^{-3} n-type) at -15 ^{o}C, using previous plot

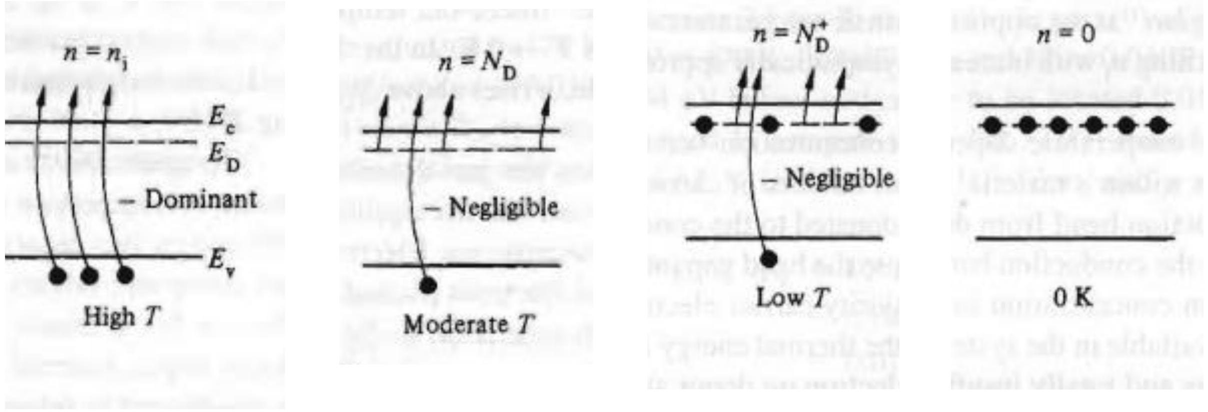
(b) n-type

Sources: E. Pop ECE 340 Slides, Textbook

Intrinsic Carrier Concentration: Temperature Regions

- _____________ **Region:** At Low T's, some donors are ionized. By ~100 K all are ionized.
- _____________ **Region:** At Moderate T's, $n_0 \approx N_d$ since all donors are ionized and one e$^-$ obtained for each donor atom
- ___________ **Region:** At High T's, # of intrinsic carriers is very high and exceeds ____





Sources: E. Pop ECE 340 Slides, Textbook

Compensation

- So far, we have assumed material is doped either n-type of p-type. At moderate temperatures:
  - $n_0 \approx$
  - $p_0 \approx$
- Sometimes we dope semiconductors with both ______ and _________. This I called compensation.
- You can even start with p-type material and convert a portion of it ______ by adding enough donors
- This is a technique frequently employed to make complex devices.
- Essentially, an acceptor can effectively negate the effect of a donor!

Sources: E. Pop ECE 340 Slides, Textbook

Charge Neutrality

- What if we introduce $N_d = N_a$?
  - The material once again becomes________________ and $n_0 \approx p_0 \approx$ _____.

- So far we have seen four types of charged species in a semiconductor:
  - Electrons
  - Holes
  - ________ donor ions
  - ________ acceptor ions

$$n + N_a = p + N_d$$

- In general, this is because we must have charge neutrality in the material
- Any semiconductor material is electrostatically ________
  - Positive charge = negative charge

Sources: E. Pop ECE 340 Slides, Textbook

Carrier Concentrations in Compensated Semiconductors

- The more detailed equations for finding the carrier concentrations given values for N_a, N_d:

$$n = \frac{N_d - N_a}{2} + \left[\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2\right]^{1/2}$$

$$p = \frac{N_a - N_d}{2} + \left[\left(\frac{N_a - N_d}{2}\right)^2 + n_i^2\right]^{1/2}$$

- How do these simplify when $N_d \gg N_a$?

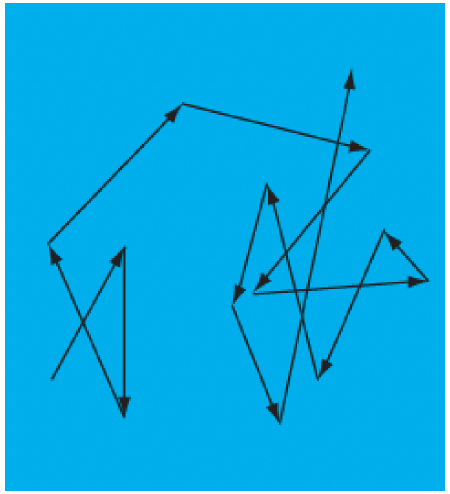
$$n = N_d \quad \text{and} \quad p = n_i^2 / N_d$$

- When is the >> approximation valid?

Sources: E. Pop ECE 340 Slides, C. Hu, *Modern Semiconductor Devices for Integrated Circuits*

Carrier Movement: No E-field

- So far, we've examined the effect of doping and temperature on carrier concentrations. We have assumed there is no ____________
- We know carriers are 'free' to move around. Charge carriers are in constant motion. What does this look like?



- Instantaneous velocity is given by thermal energy:

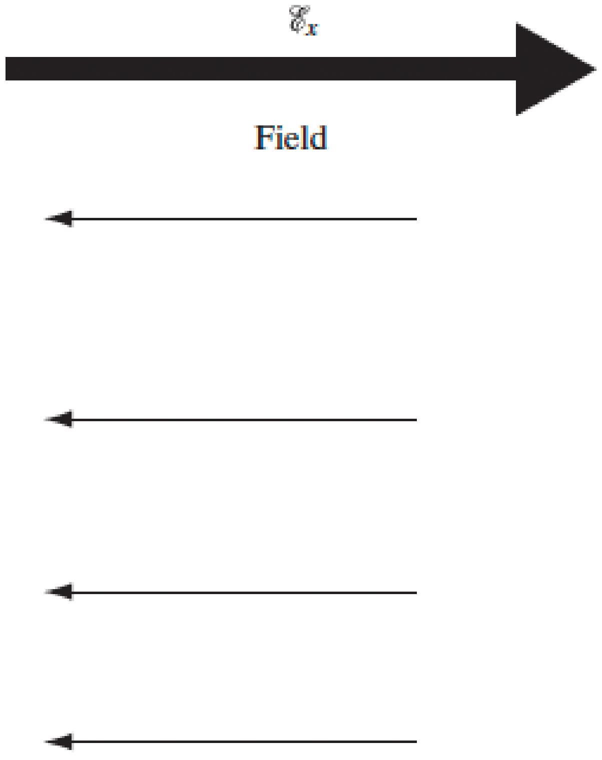
$$v_{th} = \sqrt{\frac{3kT}{m^*}}$$

- What is the *net* motion of carriers? _____
- There is no preferred __________ of motion for a group of carriers and no net current flow.
- Mean free time between collisions ~________. Mean free distance a few tens of ____.

Sources: E. Pop ECE 340 Slides, Textbook

Carrier Movement: With E-field

- Typically, we need an E-field to make use of semiconductors in devices



- Let's imagine we apply an E-field in the __-direction

- What force do e^- experience?

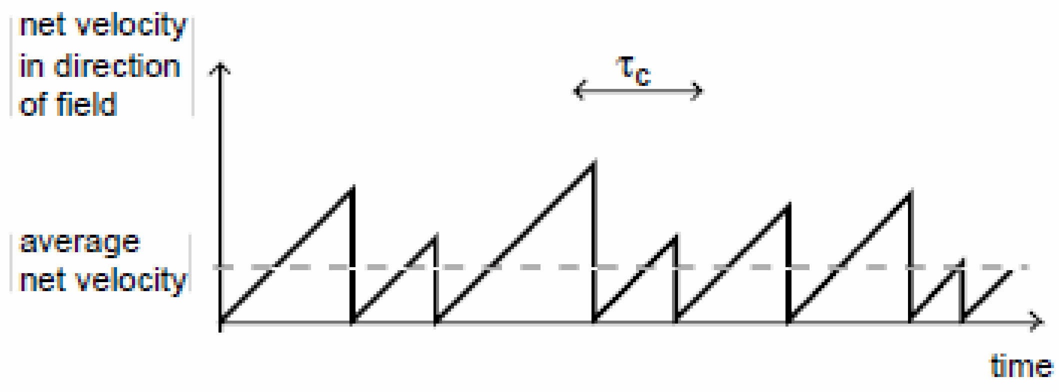
$$F = \_\_\_\_\_$$

- Is there net motion of e^-?

Sources: Textbook

Drift

- Drift is the motion of charge carriers caused by an _______________
- Individual e⁻ velocity is _____________, but the ________ velocity is non-zero



- We can assume a mean free time between collisions,_____, and that each e⁻ loses its entire drift momentum, ___, after each collision:

$$p = m_n^* v_n = -qE\tau_C$$

$$v_n = -\frac{qE\tau_C}{m_n^*}$$

Sources: E. Pop ECE 340 Slides, C. Hu, *Modern Semiconductor Devices for Integrated Circuits*

Drift Velocity and Mobility

- The drift velocity is usually written with a proportionality constant,

$$\boxed{v_n = -\mu_n E}$$

- And for holes,

$$\boxed{v_p = \mu_p E}$$

- We call this proportionality constant the ______________

$$\boxed{\mu_n = -\frac{q\tau_C}{m_n^*}} \qquad \boxed{\mu_n = \frac{q\tau_C}{m_p^*}}$$

- Describes the ________________________ in a semiconductor.
- Very important quantity!
- Units?

Sources: E. Pop ECE 340 Slides, C. Hu, *Modern Semiconductor Devices for Integrated Circuits*

Mobility

- What are the roles of τ_C and m^*?
  - If m↓ “lighter” particle means μ…
  - If $\boldsymbol{\tau}_{\mathrm{c}}$↑ means longer time between collisions, so μ…

- Mobilities at room temperature in ________ doped semiconductors:

TABLE 2–1 • Electron and hole mobilities at room temperature of selected lightly doped semiconductors.

	Si	Ge	GaAs	InAs
μ_n (cm^2/V·s)	1400	3900	8500	30, 000
μ_p (cm^2/V·s)	470	1900	400	500

- High mobility is desired in devices --> translates to ____________(frequencies)

Sources: E. Pop ECE 340 Slides, C. Hu, *Modern Semiconductor Devices for Integrated Circuits*

Problem

Given $\mu_p = 470\ cm^2/V \cdot s$ for Si, what is the hole drift velocity at $E = 10^3\ V/cm$? What is $\boldsymbol{\tau}_\mathbf{c}$ and what is the average distance traveled between collisions, i.e., the **mean free path**? Assume 300 K in the dark.

Sources: C. Hu, *Modern Semiconductor Devices for Integrated Circuits*

Mobility and Scattering

- What are the roles of τ_C and m^*?
  - If m↓ “lighter” particle means μ…
  - If $\boldsymbol{\tau_c}$↑ means longer time between collisions, so μ…

- Mobilities at room temperature of lightly doped semiconductors:

TABLE 2–1 • Electron and hole mobilities at room temperature of selected lightly doped semiconductors.

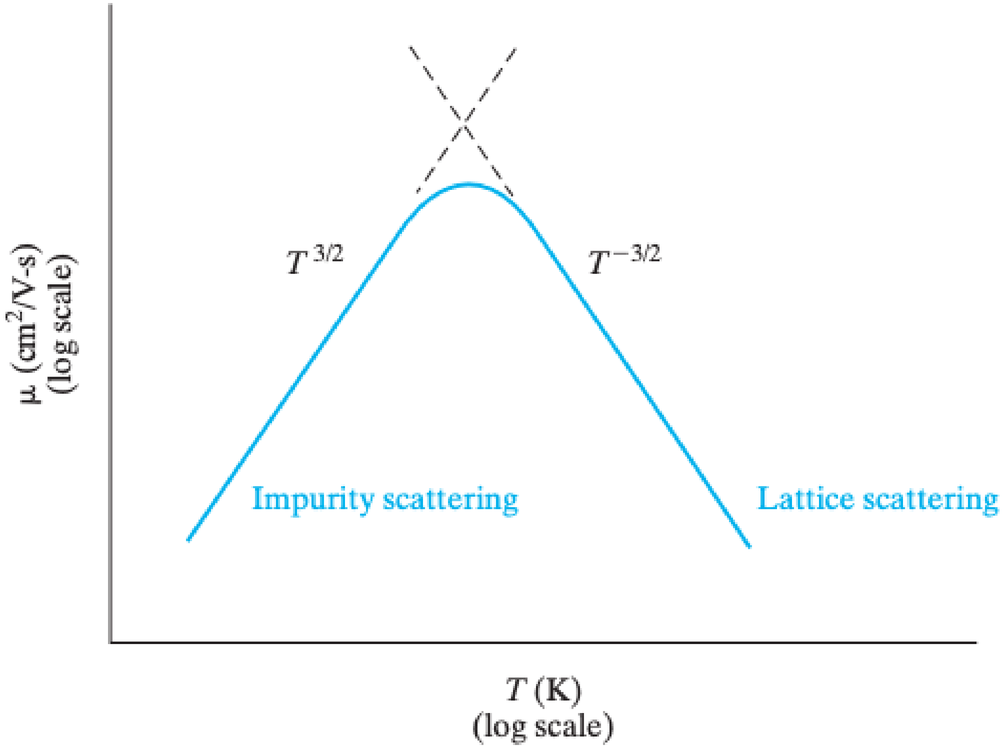
	Si	Ge	GaAs	InAs
μ_n (cm^2/V·s)	1400	3900	8500	30, 000
μ_p (cm^2/V·s)	470	1900	400	500

- High mobility is desired in devices --> translates to higher speeds (frequencies)

Sources: E. Pop ECE 340 Slides, C. Hu, *Modern Semiconductor Devices for Integrated Circuits*

Effects of Temperature on Mobility

- What does $\boldsymbol{\mu}$ (through $\boldsymbol{\tau}_\mathrm{c}$) depend on?
  - ________/lattice scattering (host lattice, like Si)
  - ______________ (dopant atom) scattering



- How is scattering effected by temperature?
  - Lattice scattering ___________ with increased temperature, and mobility ___________
  - Impurity scattering _________ with increased temperature, and mobility ________________

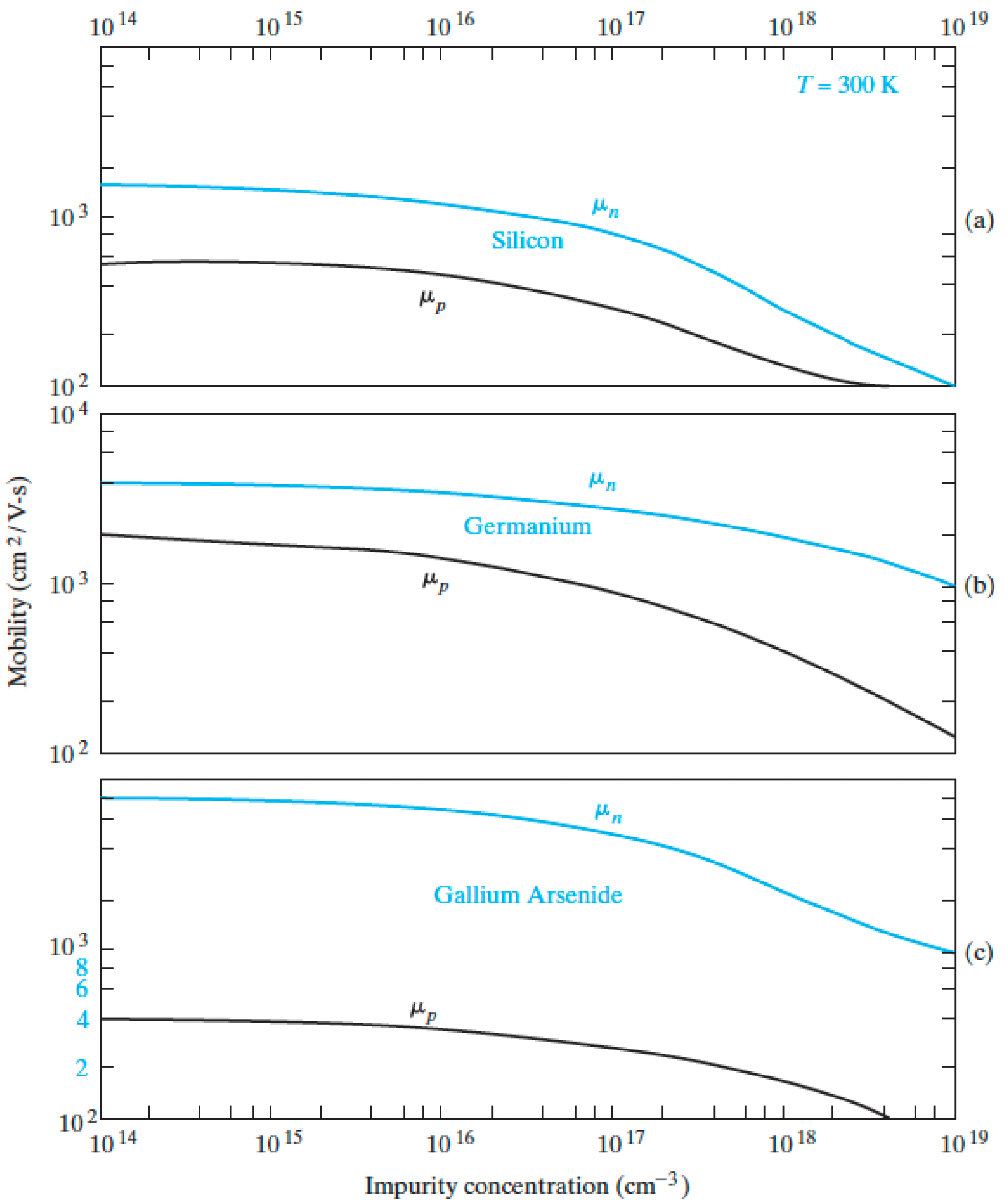
- Strongest scattering (e.g. lowest mobility) dominates total mobility:

$$\frac{1}{\mu} = \frac{1}{\mu_I} + \frac{1}{\mu_L} + \cdots$$

Sources: E. Pop ECE 340 Slides, Textbook

Effects of Doping on Mobility

- We expect the mobility to decrease with total impurities $(N_a + N_d)$
- Why? Increased ________________ scattering!
- As the concentration of dopants increases, the effects of impurity scattering are felt at higher temperatures
- In reality, mobility also depends on ____________________



Sources: Textbook

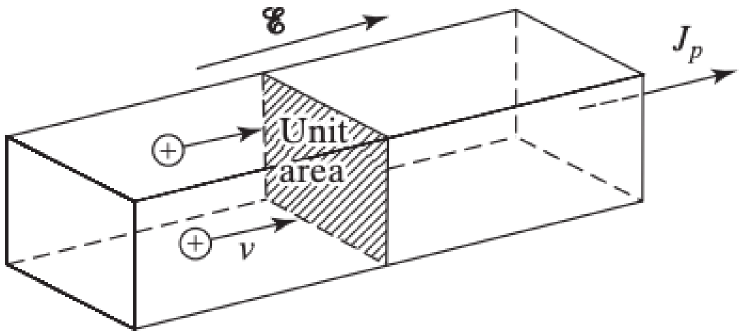
Drift Current Components

- Recall: we have a net drift velocity of carriers.
- This is ______________! Now, we can calculate current density, ___, for devices!

$$\boxed{J_n^{drift} = -qnv_{dn} = qn\mu_n E}$$

$$\boxed{J_p^{drift} = +qpv_{dp} = qp\mu_p E}$$

- So what is drift current density proportional to?
  - Carrier concentration
  - Carrier drift velocity
  - Carrier charge



A p-type semiconductor bar demonstrating the concept of current density.

- Current density, J, is the charge per second crossing a unit area plane normal to the direction of current flow (_________)

Sources: C. Hu, *Modern Semiconductor Devices for Integrated Circuits*

Total Drift Current and Conductivity

- The total drift current density is the _______ of the e^- and h^+ components:

$$J_{\text{drift}} = J_{n,\text{drift}} + J_{p,\text{drift}} = (qn\mu_n + qp\mu_p)\mathscr{E}$$

- Note: This assumes **low E-fields** (typically <______ V/cm in Si)

- The quantity in parenthesis is called the _______________,

$$\boxed{\sigma = qn\mu_n + qp\mu_p}$$

- Recall: large ratio between majority and minority carriers
  - Usually _____________________ in σ

Sources: Textbook

Resistivity

- Drift current density can therefore be written,

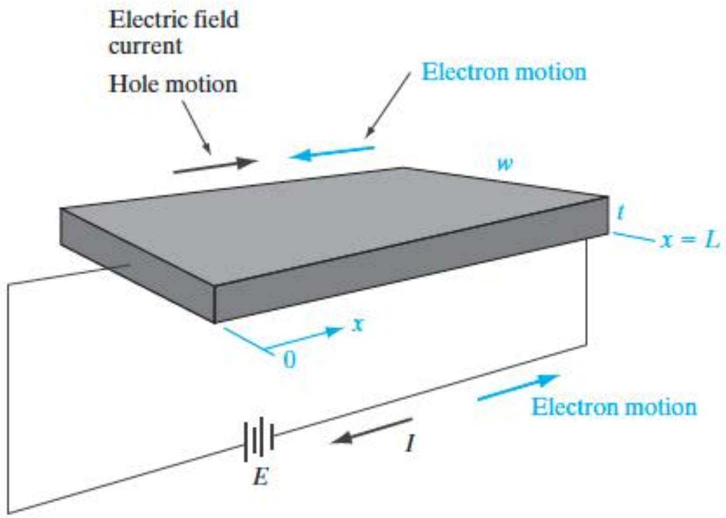
$$\boxed{J = \sigma E = \frac{E}{\rho}}$$

Ohm's law: current density is directly proportional to electric field!

- ρ is the resistivity (i.e. the ________ of conductivity),

$$\boxed{\rho = \frac{1}{\sigma} = \frac{1}{q(n\mu_n + p\mu_p)}}$$

Units: Ω-cm



- What about when n >> p? (n-type doped sample)
- What about when n << p? (p-type doped sample)

- We can find the total resistance (Ω) of a bar of semiconductor with a given width, length and thickness (__________):

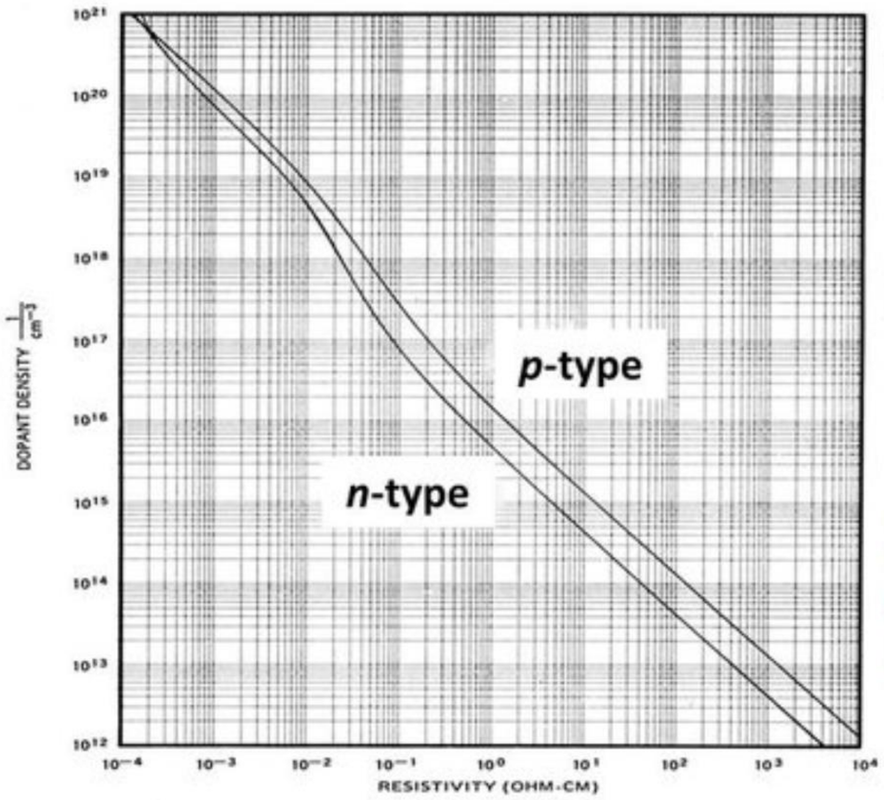
$$\boxed{R = \frac{\rho L}{wt} = \frac{L}{wt}\frac{1}{\sigma}}$$

Sources: Textbook

Resistivity Dependence on Doping

- Experimentally, for Si at room temperature we get the plot below

- We have ______________________________ through doping!



For n-type material:

$$\rho \cong \frac{1}{qn\mu_n}$$

For p-type material:

$$\rho \cong \frac{1}{qp\mu_p}$$

Note: This plot (for Si) does not apply to compensated material (doped with both acceptors and donors).

Sources: ECE 130/230M Slides